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Research Article

On the global solution of a fuzzy linear system

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Abstract

The global solution of a fuzzy linear system contains the crisp vector solution of a real linear system. So discussion about the global solution of a $n \times n$ fuzzy linear system $A\tilde{x} = \tilde{b}$ with a fuzzy number vector b in the right hand side and crisp a coefficient matrix A is considered. The advantage of the paper is developing a new algorithm to find the solution of such system by considering a global solution based upon the concept of a convex fuzzy numbers. At first the existence and uniqueness of the solution are introduced and then the related theorems and properties about the solution are proved in details. Finally the method is illustrated by solving some numerical examples.

Keywords: Fuzzy linear system, Algebraic solution, Fuzzy number.

1 Introduction

Fuzzy linear systems have many applications in science, such as control problems, information, physics, statistics, engineering, economics, finance and even social science. In the 1990s, Buckley et al. [11, 12, 13] investigated the mentioned systems in analytical form. Subsequently, Friedman et al. [14] considered a fuzzy linear system as follows,

$$\begin{cases} a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 + \ldots + a_{1n}\tilde{x}_n = \tilde{b}_1 \\ a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + \ldots + a_{2n}\tilde{x}_n = \tilde{b}_n \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ a_{n1}\tilde{x}_1 + a_{n2}\tilde{x}_2 + \ldots + a_{nn}\tilde{x}_n = \tilde{b}_n \end{cases} \tag{1.1}$$

where the coefficient matrix $A = (a_{ij})$ is a crisp matrix and $\tilde{b} = (b_i)$ is a fuzzy vector for $1 \leqslant i, j \leqslant n$. They used the embedding method and replaced the original fuzzy linear system by a crisp linear system with a nonnegative coefficient matrix S, which is singular even if A is nonsingular. They also presented conditions for the existence of a unique fuzzy solution to the system. In [3, 4], Allahviranloo has investigated the various numerical methods (Jacobi, Gauss Seidel) for solving such fuzzy linear systems for the first time. Also he proposed the Adomian and Homotopy methods for solving fuzzy linear systems in [7, 9]. The several different numerical techniques for solving them are proposed in [1, 5, 6]. Also in [2] Allahviranloo et. al. have shown that the proposed method by [14] has no weak solution generally. Recently, he and Salahshour in [10] proposed a simple and practical method to obtain fuzzy

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symmetric solutions of fuzzy linear systems. The algebraic solution of such systems and its properties are discussed in [8]. Also Ghanbari and his colleague in [15] have proposed an approach for computing the general compromised solution of an *L-R* fuzzy linear system by use of a ranking function when the coefficient matrix is a crisp $m \times n$ matrix. Zengfeng et. al in [19], worked on the perturbation analysis of fuzzy linear systems. Wang et. al in [18], considered Jacobi and Gauss Seidel iteration methods for solving the fuzzy linear system. Summarised the structure of the paper is as follows:
In section 2, some basic definitions and notions about fuzzy concepts are brought. In section 3, we are going to represent the algebraic solution and its properties. In section 4 the method and main results are discussed. In section 5, based on the proposed method, an algorithm for solving some numerical examples is designed. Finally, conclusions are drawn in section 6.

2 Preliminaries

In this section some basic definitions and notations are brought.

Definition 2.1. *A fuzzy number $\tilde{x}$ is shown as an ordered pair of functions $\tilde{x} = \big(\underline{x}(r), \bar{x}(r)\big)$ where:*
i) $\underline{x}(r)$ is a left-continuous bounded monotonic increasing function.
ii) $\bar{x}(r)$ is a left-continuous bounded monotonic decreasing function.
iii) $\underline{x}(r) \leq \bar{x}(r), \;\; r \in [0,1]$.

The set of fuzzy numbers is shown in the form of E^1.

Definition 2.2. *If $\tilde{x} \in E^1$ then the support is defined as follows:*

$$supp\ \tilde{x} = \{\overline{x \in R \mid \; \mu_{\tilde{x}}(x) > 0}\} = [\underline{x}(0), \bar{x}(0)].$$

And if $\tilde{x} \in E^n$ then:

$$supp\ \tilde{x} = \prod_{j=1}^{n} [\underline{x}_j(0), \bar{x}_j(0)].$$

Definition 2.3. *A fuzzy number in $\tilde{x} \in E^1$ is called convex if all $r-$level sets are convex for each r. Also a fuzzy number vector $\tilde{x} \in E^n$ is called convex when $\prod_{j=1}^{n} [\underline{x}_j(r), \bar{x}_j(r)]$ is a convex polygon.*

Definition 2.4. *A fuzzy number vector is shown as $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)^t \in E^n$ in which $\tilde{x}_i$ is a fuzzy number in E^1. It could be written as:*

$$\tilde{x} = \big(\underline{x}(r), \bar{x}(r)\big) = \Big(\big(\underline{x}_1(r), \ldots, \underline{x}_n(r)\big)^t, \big(\bar{x}_1(r), \ldots, \bar{x}_n(r)\big)^t\Big) \tag{2.2}$$

where for $0 \leq r \leq 1$,

$$\big(\underline{x}_1(r), \ldots, \underline{x}_n(r)\big)^t = \min\{ \quad u = (u_1, u_2, \ldots, u_n)^t \mid$$
$$u \in \textstyle\prod_{j=1}^{n} [\underline{x}_j(r), \bar{x}_j(r)], b \in \prod_{i=1}^{n} [\underline{b}_i(r), \overline{b}_i(r)], Au = b\},$$

$$\big(\bar{x}_1(r), \ldots, \bar{x}_n(r)\big)^t = \max\{ \quad u = (u_1, u_2, \ldots, u_n)^t \mid$$
$$u \in \textstyle\prod_{j=1}^{n} [\underline{x}_j(r), \bar{x}_j(r)], b \in \prod_{i=1}^{n} [\underline{b}_i(r), \overline{b}_i(r)], Au = b\}.$$

Also based on the extension principle, the membership function of an arbitrary vector $x = (x_1, \ldots, x_n)^t \in R^n$ is defined as the following:

$$\mu_{\tilde{x}}(x) = \min_{1 \leq j \leq n} \{\mu_{\tilde{x}_j}(x_j)\}$$

in which $\mu_{\tilde{x}_j}(x), \; j = 1, 2, \ldots, n$ is the membership function of fuzzy number $\tilde{x}_j$.

3 Fuzzy linear system

Consider the $n \times n$ fuzzy linear system (1.1). It can be written in matrix form as follows,

$$A\tilde{x} = \tilde{b} \tag{3.3}$$

We denote the set of all crisp solutions of system (3.3) by $\chi_{\exists}$. So then,

$$\chi_{\exists} = \{x \in R^n \mid \ \exists b \in \tilde{b} \ ; \ Ax = b\}.$$

Definition 3.1. *The vector $\tilde{x} \in E^n$ will be called an algebraic solution of system (3.3) if:*

$$\sum_{j=1}^{n} a_{ij}\tilde{x}_j = \tilde{b}_i \Rightarrow \sum_{j=1}^{n} a_{ij}[\underline{x}_j(r), \overline{x}_j(r)] = [\underline{b}_j(r), \overline{b}_j(r)], \ \ i = 1, 2, \ldots, n. \tag{3.4}$$

Note 1: If the system (3.3) has the algebraic solution $\tilde{x}$, then $\tilde{x} \subseteq \chi_{\exists}$.
Note 2: If we just deal with the algebraic solution, we should consider the following **4** cases:

1. From (3.4) it is clear that the algebraic solution is obtained by the exact equality between two fuzzy numbers.

$$\tilde{a}, \tilde{b} \in E^1, \ \ \tilde{a} = \tilde{b} \ \ i.e \ \ \underline{a}(r) = \underline{b}(r), \ \ \overline{a}(r) = \overline{b}(r)$$

Using the definition 3.1, each equation is transformed to the following equations:

$$\begin{aligned} \sum_{j=1}^{n} \underline{a_{ij}x_j(r)} &= \underline{b}(r), \quad i = 1, 2, \ldots n \\ \sum_{j=1}^{n} \overline{a_{ij}x_j(r)} &= \overline{b}(r), \quad i = 1, 2, \ldots n \end{aligned}$$

So, two $n \times n$ crisp systems are produced.

2. In the proposed methods to find the algebraic solution, it is necessary that $2n \times 2n$ crisp system or two $n \times n$ crisp systems are solved.

3. To find the algebraic solution, the interval arithmetic is employed. Since the calculate operations on intervals are based on the extension principle, it causes the extension of the width of intervals. Therefore, in application there is usually no algebraic solution. See [19, 21] for more information.

4. Since the algebraic solution is a subset of $\chi_{\exists}$, it may does not include all the crisp vectors. For description see the following example:

Example 3.1. *[14], Consider the 2×2 fuzzy linear system as following:*

$$\begin{cases} x_1 - x_2 = (r, 2 - r) \\ x_1 + 3x_2 = (4 + r, 7 - 2r) \end{cases} \tag{3.5}$$

The solution of the system is as:

$$\begin{aligned} \underline{x}_1(r) = 1.375 + 0.625r \quad &, \quad \overline{x}_1(r) = 2.875 - 0.875r \\ \underline{x}_2(r) = 0.875 + 0.125r \quad &, \quad \overline{x}_2(r) = 1.375 - 0.375r \end{aligned}$$

It is seen that, if $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)^t$ and $\tilde{b} = (\tilde{b}_1, \tilde{b}_2)^t$ then:

$$\begin{aligned} supp \ \tilde{x} &= [1.375, 2.875] \times [0.875, 1.375] \\ supp \ \tilde{b} &= [0, 2] \times [4, 7] \end{aligned}$$

Now, if we choose vector $\tilde{b} = (0.1, 4.1) \in supp \ \tilde{b}$ then we have:

$$\begin{cases} \hat{x}_1 - \hat{x}_2 = 0.1 \\ \hat{x}_1 + 3\hat{x}_2 = 4.1 \end{cases} \Rightarrow \hat{x} = (1.1, 1)^t$$

It is clear that, $\hat{x} \notin supp \ \tilde{x}$ and thus, $\mu_{\tilde{x}}(\hat{x}) = 0$.

4 The proposed method

In this section we are going to introduce the unique algebraic solution of system (3.3). Let A be nonsingular.

Definition 4.1. *The vector* $\tilde{X} = \left(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n\right)^t$ *in which* $\tilde{x}_i = (\underline{x}_i(r), \overline{x}_i(r)), i = 1, \ldots, n$ *is called a global solution of system (3.3) whenever for* $r \in [0,1]$*:*

$$\underline{x}_i(r) = \inf \left\{x_i(r) \mid x(r) = \left(x_1(r), \ldots, x_n(r)\right)^t \in \chi_{\exists}\right\}$$
$$\overline{x}_i(r) = \sup \left\{x_i(r) \mid x(r) = \left(x_1(r), \ldots, x_n(r)\right)^t \in \chi_{\exists}\right\}$$

Theorem 4.1. *If in system (3.3), matrix A is nonsingular, then there is a unique global fuzzy number vector solution.*

Proof. The proof of uniqueness is clear and for existence it is structural. We define two sets of vectors as the following for arbitrary and fixed $0 \le r \le 1$:

$$I(r) = \left\{v(r) \in R^n \mid v(r) = \left(v_1(r), \ldots, v_n(r)\right)^t,\ v_j(r) \in \{\underline{b}_j(r), \overline{b}_j(r)\}\right\}$$

$$\chi(r) = \left\{x(r) \in R^n \mid x(r) = \left(x_1(r), \ldots, x_n(r)\right)^t,\ Ax(r) = v(r) \in I(r)\right\}$$

The set $I(r)$ has 2^n elements. Thus the set $\chi(r)$ is obtained by solving 2^n crisp systems. It is clear that, in definition 4.1, "inf" and "sup" is replaced by "min" and "max", as the following. So for $j = 1, \ldots, n$,

$$\underline{x}_j(r) = \min_{1 \le k \le 2^n} \left\{x_j^{(k)}(r) \mid x^{(k)} = (x_1^{(k)}, \ldots, x_n^{(k)})^t,\ x^{(k)} \in \chi(r)\right\} \tag{4.6}$$

$$\overline{x}_j(r) = \max_{1 \le k \le 2^n} \left\{x_j^{(k)}(r) \mid x^{(k)} = (x_1^{(k)}, \ldots, x_n^{(k)})^t,\ x^{(k)} \in \chi(r)\right\} \tag{4.7}$$

in which, $0 \le r \le 1$ is arbitrary and fixed.
Now, we show that $\tilde{x}_j = \left(\underline{x}_j(r), \overline{x}_j(r)\right)$ for $r \in [0,1]$, is a fuzzy number. Considering structural proof, it is clear that $\underline{x}_j(r) \le \overline{x}_j(r)$. Assume that $A^{-1} = [t_{ij}]_{i,j=1}^n$. For any r, the vector $v^{(k)}(r) \in I(r)$ corresponds to the vector $x^{(k)}(r) \in \chi(r)$, and we have:

$$\begin{aligned} Ax^{(k)}(r) &= v^{(k)}(r) \Rightarrow x^{(k)}(r) = A^{-1}v^{(k)}(r) \\ \Rightarrow x_i^{(k)}(r) &= \sum_{j=1}^{n} t_{ij} v_j^{(k)}(r) \\ &= \sum_{t_{ij} \ge 0}^{n} t_{ij} v_j^{(k)}(r) + \sum_{t_{ij} < 0}^{n} t_{ij} v_j^{(k)}(r) \end{aligned} \tag{4.8}$$

Using (4.6) and (4.7) in (4.8), we have:

$$\begin{aligned} \underline{x}_i(r) &= \min_{1 \le k \le 2^n} x_i^{(k)}(r) = \min_{1 \le k \le 2^n} \sum_{j=1}^{n} t_{ij} v_j^{(k)}(r) \\ &= \sum_{t_{ij} \ge 0}^{n} t_{ij} \underline{b}_j(r) + \sum_{t_{ij} < 0}^{n} t_{ij} \overline{b}_j(r), \quad r \in [0,1] \end{aligned} \tag{4.9}$$

$$\begin{aligned} \overline{x}_i(r) &= \max_{1 \le k \le 2^n} x_i^{(k)}(r) = \max_{1 \le k \le 2^n} \sum_{j=1}^{n} t_{ij} v_j^{(k)}(r) \\ &= \sum_{t_{ij} \ge 0}^{n} t_{ij} \overline{b}_j(r) + \sum_{t_{ij} < 0}^{n} t_{ij} \underline{b}_j(r), \quad r \in [0,1] \end{aligned} \tag{4.10}$$

Since $\tilde{b}_j$ is a fuzzy number then, from the coefficients of $\overline{b}_j(r), \underline{b}_j(r)$ in (4.9), $\underline{x}_i(r)$ is a bounded monotonic left-continuous increasing function. In the same way, we conclude from (4.10), $\overline{x}_i(r)$ is a bounded monotonic left-continuous decreasing function. So, $\tilde{x}_i$ is a fuzzy number. Consequently, vector $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ where $\tilde{x}_i,\ i = 1, \ldots, n$ is obtained from (4.9) and (4.10) is a fuzzy number vector and the proof is completed. □

Proposition 4.1. *i) If $\tilde{x} \in E^n$ is as a global solution of (3.3) then:*

$$\sum_{j=1}^{n} a_{ij}\tilde{x}_j(1) = \tilde{b}_i(1), \quad i = 1,\dots,n \tag{4.11}$$

ii) If $\tilde{x} = (\tilde{x}_1,\dots,\tilde{x}_n)^t$ and $\tilde{y} = (\tilde{y}_1,\dots,\tilde{y}_n)^t$ are global solutions and algebric solutions of system (3.3) respectively, then:

$$\tilde{y}(1) = \tilde{x}(1) \;\; i.e \;\; \tilde{y}_j(1) = \tilde{x}_j(1), \; j = 1,2,\dots,n$$

Proof. i) Considering (4.9) and (4.10) and since $\tilde{b}_j(1) = \underline{b}_j(1) = \overline{b}_j(1)$ therefore:

$$\tilde{x}_j(1) = \sum_{j=1}^{n} t_{ij}\tilde{b}_i(1), \quad i = 1,\dots,n$$

So the proof is completed.
ii) Since $\tilde{y}$ is an algebric solution thus:

$$\sum_{j=1}^{n} a_{ij}\tilde{y}_j(r) = \tilde{b}_i(r), \quad i = 1,\dots,n, r \in [0,1]$$

So by choosing $r = 1$ we have:

$$\sum_{j=1}^{n} a_{ij}\tilde{y}_j(1) = \tilde{b}_i(1), \quad i = 1,\dots,n$$

but this system equals to system (4.11). Consequently the proposition (ii) is true. □

Note 3: Cases (4.9) and (4.10) show that the global solution can be obtained by solving a $n \times n$ crisp system. Let us consider the subject from another point of view. Since $\tilde{b}_j = \left(\underline{b}_j(r), \overline{b}_j(r)\right), \; j = 1,\dots,n$ are convex fuzzy numbers so we can change system (3.3) in the form of two $n \times n$ crisp systems with the parameter values on the right hand side. In other words:

$$\sum_{j=1}^{n} a_{ij}x_j(r) = \lambda_i\overline{b}_i(r) + (1-\lambda_i)\underline{b}_i(r), \quad i = 1,\dots,n \tag{4.12}$$

in which $\lambda_i \in [0,1]$. By considering $A^{-1} = (t_{ij})$ from (4.12), we conclude that:

$$x_i(r) = \sum_{j=1}^{n} t_{ij}\left(\lambda_j\overline{b}_j(r) + (1-\lambda_j)\underline{b}_j(r)\right), \quad i = 1,\dots,n \tag{4.13}$$

By choosing $r = 0$, the system (4.12) includes all crisp systems that are produced from systems (3.3) and (4.13). It is the general form of each crisp vector solution of system (3.3).
Now we are going to have (4.9) and (4.10) in different but simple forms. To this end,

$$\begin{aligned} \underline{x}_i(r) &= \textstyle\sum_{j=1}^{n} t_{ij}\left(\lambda_{ij}\overline{b}_j(r) + (1-\lambda_{ij})\underline{b}_j(r)\right) \\ \overline{x}_i(r) &= \textstyle\sum_{j=1}^{n} t_{ij}\left((1-\lambda_{ij})\overline{b}_j(r) + \lambda_{ij}\underline{b}_j(r)\right) \end{aligned} \tag{4.14}$$

where

$$\lambda_{ij} = \begin{cases} 1; & t_{ij<0} \\ 0; & t_{ij\geq 0} \end{cases}, \; j = 1,2,\dots,n \tag{4.15}$$

5 Algorithm

In this section based on (4.9), (4.10) and (4.13) an algorithm is proposed for the calculation of global solution in a way that this algorithm includes simple and easy procedures for solution production. Then for the explanation of algorithm and its practical applications, two examples are solved.

Algorithm:
Step 1: Consider the system (3.3) as the entrance, then calculate matrix $A^{-1} = (t_{ij}),\ 1 \leq i, j \leq n$.
Step 2: For $i = 1, 2, \ldots, n$ let:

$$\underline{x}_i(r) = \sum_{j=1}^{n} t_{ij}\Big(\underline{\lambda}_{ij}\overline{b}_j(r) + (1-\underline{\lambda}_{ij})\underline{b}_j(r)\Big) \tag{5.16}$$

where:

$$\underline{\lambda}_{ij} = \begin{cases} 1; & t_{ij<0} \\ 0; & t_{ij\geq 0} \end{cases},\ j = 1, 2, \ldots, n \tag{5.17}$$

Step 3: In step 2, $\underline{\lambda}_{ij}$ is exchanged with $\overline{\lambda}_{ij} = 1 - \underline{\lambda}_{ij}$ and also $\underline{x}_i(r)$ is exchanged by $\overline{x}_i(r)$.
Step 4: Fuzzy number vector $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_n)^t$ where $\tilde{x}_j = (\underline{x}_j(r), \overline{x}_j(r)),\ j = 1, 2, \ldots, n$ obtained from steps 2 and 3, is the global solution.

Example 5.1. *Consider the* 2×2 *fuzzy linear system in the form of (3.5) in example 3.1. The description is as the following:*

$$\begin{aligned} x_1 - x_2 &= \lambda_1(2-r) + (1-\lambda_1)r \\ x_1 + 3x_2 &= \lambda_2(7-2r) + (1-\lambda_2)(4+r),\ \ \lambda_1, \lambda_2 \in [0,1] \end{aligned}$$

From (5.16) we have:

$$\begin{aligned} \underline{x}_1 &= \frac{1}{4}\big(3\underline{\lambda}_{11}(2-r) + 3(1-\underline{\lambda}_{11})r + \underline{\lambda}_{12}(7-2r) + (1-\underline{\lambda}_{12})(4+r)\big) \\ \underline{x}_2 &= \frac{1}{4}\big(-\underline{\lambda}_{21}(2-r) - (1-\underline{\lambda}_{21})r + \underline{\lambda}_{22}(7-2r) + (1-\underline{\lambda}_{22})(4+r)\big) \end{aligned}$$

Considering the coefficients $\underline{\lambda}_{ij}$ from (5.17), we obtain:

$$\underline{\lambda}_{11} = \underline{\lambda}_{12} = 0,\ \ \underline{\lambda}_{21} = 1,\ \ \underline{\lambda}_{22} = 0$$

Therefore, from the step 3 we will have:

$$\begin{aligned} \tilde{x}_1 &= (1+r\ ,\ 3.25-1.25r) \\ \tilde{x}_2 &= (0.5+0.5r\ ,\ 1.75-0.75r) \end{aligned}$$

So choosing $\tilde{b} = (0.1, 4.1) \in supp\ \tilde{b}$, for the obtained solution $\hat{x} = (1.1, 1)^t$, $\hat{x} \in supp\ \tilde{x}$.

Example 5.2. *(Ming. Ma, [14]) Consider the* 3×3 *fuzzy system:*

$$\begin{aligned} x_1 + x_2 + x_3 &= (r, 2-r) \\ x_1 - 2x_2 + x_3 &= (2+r, 3) \\ 2x_1 + x_2 + 3x_3 &= (-2, -1-r) \end{aligned}$$

The algebraic solution is as the following:

$$\begin{aligned} x_1 &= (-2.31+3.62r\ ,\ 4.29-3.38r) \\ x_2 &= (-0.62-0.77r\ ,\ -1.62+0.23r) \\ x_3 &= (1.08-2.15r\ ,\ -2.92+1.85r) \end{aligned}$$

It can be seen that this solution is not a fuzzy number solution (because of x_2 and x_3). In other words, the system has not any fuzzy number solution so, we have:

$$A^{-1} = \begin{pmatrix} 0.333 & 0.267 & 0.2 \\ 0.333 & -0.333 & 0 \\ 0.333 & 0.067 & -0.2 \end{pmatrix}$$

And with the help of step 2:

$$\begin{aligned} \underline{x}_1 &= 0.333\big(\underline{\lambda}_{11}(2-r)+(1-\underline{\lambda}_{11})r\big)+0.267\big(3\underline{\lambda}_{12}+(1-\underline{\lambda}_{12})(2+r)\big) \\ &\quad +0.2\big(\underline{\lambda}_{13}(-1-r)-2(1-\underline{\lambda}_{13})\big) \\ \underline{x}_2 &= 0.333\big(\underline{\lambda}_{21}(2-r)+(1-\underline{\lambda}_{21})r\big)-0.333\big(3\underline{\lambda}_{22}+(1-\underline{\lambda}_{22})(2+r)\big) \\ \underline{x}_3 &= 0.333\big(\underline{\lambda}_{31}(2-r)+(1-\underline{\lambda}_{31})r\big)+0.067\big(3\underline{\lambda}_{32}+(1-\underline{\lambda}_{32})(2+r)\big) \\ &\quad -0.2\big(\underline{\lambda}_{33}(-1-r)-2(1-\underline{\lambda}_{33})\big) \end{aligned}$$

From the coefficients $\underline{\lambda}_{ij}$ the following result is obtained:

$$\begin{aligned} \underline{\lambda}_{11}=\underline{\lambda}_{12}=\underline{\lambda}_{13}=0 \quad &, \quad \underline{\lambda}_{21}=0, \quad \underline{\lambda}_{22}=1 \\ \underline{\lambda}_{31}=\underline{\lambda}_{32}=0 \quad &, \quad \underline{\lambda}_{33}=1 \end{aligned}$$

And using step 3 we will have:

$$\begin{aligned} \tilde{x}_1 &= (0.134+0.6r\ ,\ 1.267-0.533r) \\ \tilde{x}_2 &= (-0.999+0.333r\ ,\ -0.666r) \\ \tilde{x}_3 &= (0.334+0.6r\ ,\ 1.267-0.333r) \end{aligned}$$

Therefore, fuzzy number vector $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ is the global solution of system.

6 Conclusions

As observed, the proposed method shows that if A^{-1} exists, then the fuzzy linear system always has a unique global solution in the form of a fuzzy number vector. Although in this method a simple and easy calculations obtain the global solution without employing interval arithmetics.

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